Available online at www.sciencedirect.com
SCIENCE DIRECT®
Journal of Algebra 296 (2006) 249–252
JOURNAL OF Algebra
www.elsevier.com/locate/jalgebra

Corrigendum

Corrigendum to "Minimum polynomials and lower bounds for eigenvalue multiplicities of prime-power order elements in representations of classical groups" [J. Algebra 243 (2001) 228–263]

L. Di Martino [a,*], A.E. Zalesskii [b]

[a] *Dipartimento di Matematica e Applicazioni, Università degli Studi di Milano-Bicocca, via Bicocca degli Arcimboldi 8, 20126 Milan, Italy*
[b] *School of Mathematics, University of East Anglia, Norwich, UK*

Received 29 December 2004
Available online 20 April 2005
Communicated by Jan Saxl

The statement of Theorem 1.1 of the above paper should include an additional exceptional case (case (f) below), which was missed because of a flaw in the proof of Lemma 5.4. This also requires refining of the statements of Theorems 1.2 and 5.1 and the proofs of Lemma 5.4 and Proposition 5.5. Remark (3) to Theorem 1.1 is revised accordingly. All the other statements and arguments in the paper remain valid and require no changes.

Below a correct version of the statements of Theorems 1.1, 1.2, and 5.1 is provided, together with the amended proofs of Lemma 5.4 and Proposition 5.5.

Theorem 1.1. *Let $g \in \widetilde{H}$ be a non-central element of prime-power order s modulo $Z(\widetilde{H})$, which stabilizes a non-zero totally isotropic* (*totally singular*) *subspace of V, and assume that s is coprime to p. Let h be the image of g in H, and let $\tilde{\theta}$ be an irreducible represen-*

DOI of original article: 10.1006/jabr.2001.8837.
* Corresponding author.
E-mail addresses: dimartino@matapp.unimib.it, lino.dimartino@unimib.it (L. Di Martino), zalesskii@uea.ac.uk (A.E. Zalesskii).

doi:10.1016/j.jalgebra.2005.02.006

tation of $\widetilde{H}$ over P. Then $\deg\tilde{\theta}(g) \geqslant s-k$, *where either* $k=1$, *or* $k>1$ *and* $s=k(q+1)$. *If* $\deg\tilde{\theta}(g) < s$, *then for some* $z \in Z(I(V))$ *one of the following holds*:

(a) $H = Sp(m,p)$, $m>2$, p *is an odd prime*, $s=p+1$ *and* $\operatorname{rank}(h-z)=2$.
(b) $SU(m,p) \subseteq H \subseteq U(m,p)$, $m>2$, p *is an odd prime*, $s=p+1$ *and* $\operatorname{rank}(h-z)=1$.
(c) $SU(m,q) \subseteq H \subseteq U(m,q)$, $m>2$, q *is even*, $s=q+1$ *is a prime and* $\operatorname{rank}(h-z)=1$.
(d) $SU(m,8) \subseteq H \subseteq U(m,8)$, $m>2$, $s=9$, *and* $\operatorname{rank}(h-z)=1$.
(e) $SU(m,2) \subseteq H \subseteq U(m,2)$, $m>4$, $s=9$, $k=1$ *and* $\operatorname{rank}(h-z)=3$.
(f) $SU(m,q) \subseteq H \subseteq U(m,q)$ *and* $\operatorname{rank}(h^k-z)=1$, *where* $k=s/(q+1)>1$ *and* $m \equiv 1 \pmod{k}$; *moreover*, $m-1>k$ *if* s *is odd*.

Furthermore, in each of the situations (a)–(f) *there exists a representation* θ *of* H (*hence of* $\widetilde{H}$) *such that* $\deg\theta(h)$ *is exactly* $s-k$.

Remark (3). Let $\tilde{\theta}(g^s) = \lambda \cdot \mathrm{Id}$, where $\lambda \in P$. It follows from Theorem 5.1 below that the minimum polynomial of $\tilde{\theta}(g)$ is $x^s - \lambda$ unless one of the exceptional cases holds. In cases (a) to (e) the minimum polynomial of $\tilde{\theta}(g)$ is $(x^s-\lambda)/(x-\mu)$, where $\mu \in P$ and $\mu^s = \lambda$.

The condition $m > k+1$ if s is odd in (f) is required for h to stabilize a non-zero totally isotropic subspace of V. Furthermore, the occurrence of case (f) follows immediately from cases (b), (c), and (d). Indeed, let $s = v^\gamma$, v a prime. Choose g as in one of cases (b), (c), and (d), and suppose there exists $g_1 \in \widetilde{H}$ such that $g_1^{v^i} = g$ for some i. Then $\deg\tilde{\theta}(g_1) \leqslant v^i \deg\tilde{\theta}(g) < v^i s = |g_1|$ modulo $Z(\widetilde{H})$. The representations $\tilde{\theta}$ occurring in cases (a) to (e) have been identified by Guralnick, Magaard, Saxl, and Tiep [J. Algebra 257 (2002) 297–347]. We have been informed that this is extended to case (f) in a forthcoming contribution by the same authors. It turns out that $\tilde{\theta}$ is a Weil representation and $\deg\tilde{\theta}(g) = s-k$ in this case. Thus in case (f) the minimum polynomial of $\tilde{\theta}(g)$ is $(x^s-\lambda)/(x^k-\alpha)$, where $\alpha \in P$ and $\alpha^{q+1} = \lambda$.

Theorem 1.2. *Let* $g \in \widetilde{H}$ *be a semisimple element of prime-power order* s *modulo* $Z(\widetilde{H})$, *let* h *be the image of* g *in* H, *and set* $G = \langle g\rangle$, $G_0 = \langle g^s\rangle$. *Assume that* $\tilde{\theta}$ *is an irreducible representation of* $\widetilde{H}$ *over* P, *and let* $\tilde{\theta}_{|G_0} = \nu \cdot \mathrm{Id}$, *with* $\nu \in \mathrm{Irr}_P(G_0)$. *If* $m > 8s$, *then* $\tilde{\theta}_{|G}$ *contains the direct sum of at least* $[|F|^{(m-6s)/2}]$ *copies of the induced representation* ν^G, *unless one of the cases* (a)–(f) *of Theorem* 1.1 *holds. In each of cases* (a)–(f) $\tilde{\theta}_{|G}$ *contains the direct sum of at least* $[|F|^{(m-4s)/2}]$ *copies of the quotient of* ν^G *by* μ^G, *where* μ *is an irreducible representation of* G^k, $k=1$ *in cases* (a)–(e) *and* $k = s/(q+1) > 1$ *in case* (f).

Theorem 5.1. *Let* $g \in \widetilde{H}$ *be an element of prime power order* s *modulo* $Z(\widetilde{H})$, *and assume that* s *is coprime to* p. *Further, assume that* g *lies in a proper parabolic subgroup* $\widetilde{S}$ *of* $\widetilde{H}$ *such that* $\widetilde{U} = O_p(\widetilde{S})$ *is non-abelian. Set* $G = \langle g\rangle$, $G_0 = \langle g^s\rangle$, *and* $h = \sigma(g)$. *Assume that* $\tilde{\theta}$ *is an irreducible representation of* $\widetilde{H}$ *over* P, *and let* $\tilde{\theta}_{|G_0} = \zeta \cdot \mathrm{Id}$, *where* $\zeta \in \mathrm{Irr}_P(G_0)$. *Then* $\tilde{\theta}_{|G}$ *contains the direct sum of at least* $\max\{1, [|F|^{(m-6s)/2}]\}$ *copies of* ζ^G, *unless there exists* $z \in Z(I(V))$ *such that one of the following holds*:

(a) $H = Sp(m, p)$, $m > 2$, p *is an odd prime*, $s = p + 1$, *and* $\mathrm{rank}(h - z) = 2$.
(b) $SU(m, p) \subseteq H \subseteq U(m, p)$, $m > 2$, p *is an odd prime*, $s = p+1$, *and* $\mathrm{rank}(h-z) = 1$.
(c) $SU(m, q) \subseteq H \subseteq U(m, q)$, $m > 2$, q *is even*, $s = q+1$ *is a prime, and* $\mathrm{rank}(h-z) = 1$.
(d) $SU(m, 8) \subseteq H \subseteq U(m, 8)$, $m > 2$, $s = 9$, *and* $\mathrm{rank}(h - z) = 1$.
(e) $SU(m, 2) \subseteq H \subseteq U(m, 2)$, $m > 4$, $s = 9$, *and* $\mathrm{rank}(h - z) = 3$.
(f) $SU(m, q) \subseteq H \subseteq U(m, q)$ *and* $\mathrm{rank}(h^k - z) = 1$, *where* $k = s/(q + 1) > 1$ *and* $m \equiv 1 \pmod k$; *moreover*, $m - 1 > k$ *if* s *is odd.*

In cases (a) *to* (e) $\tilde{\theta}_{|G}$ *contains the direct sum of at least* $\max\{1, [|F|^{(m-4s)/2}]\}$ *copies of a quotient of* ζ^G *by a* 1*-dimensional representation. In case* (f) $\tilde{\theta}_{|G}$ *contains the direct sum of at least* $\max\{1, [|F|^{(m-4s)/2}]\}$ *copies of a quotient of* ζ^G *by* μ^G, *where* μ *is an irreducible representation of* $G^k = \langle g^k \rangle$.

Lemma 5.4. *Set* $c := \max\{1, [|F|^{(m-2d)/2-2l}]\}$. *Assume* $k > 1$ *and let* $\zeta_\kappa \in \mathrm{Irr}_P(N_0)$ *be such that* $N_{0|T_\kappa} = \zeta_\kappa \cdot \mathrm{Id}$. *Then* $\zeta_\kappa = \zeta$, *where* $\tilde{\theta}_{|Z(\widetilde{H})} = \zeta \cdot \mathrm{Id}$, *and the following holds. Either*

(1) $N_{1|T_\kappa}$ *contains a direct sum of at least* c *copies of* ζ^{N_1}; *or*
(2) $H' \cong SU(m, q)$ *and* $\mathrm{rank}(h^k - z) = 1$ *for some* $z \in Z(I(V))$, *where* $k = s/(q + 1) > 1$ *and* $m \equiv 1 \pmod k$; *moreover* $m - 1 > k$ *if* s *is odd. In this case* $N_{1|T_\kappa}$ *contains a direct sum of at least* c *copies of a quotient* ζ^{N_1}/μ, *where* $\mu \in \mathrm{Irr}_P(N_1)$.

Proof. As $N_0 \subset Z(\widetilde{H}) \subset \widetilde{S}$ and ϕ is a constituent of $\tilde{\theta}_{|\widetilde{S}}$, the claim that $\zeta_\kappa = \zeta$ is obvious. Let λ be an irreducible constituent of T_κ, viewed as a $P\widetilde{J}_1$-module. Clearly λ is non-trivial on U'. Thus it follows from Proposition 4.1 that $N_{1|T_\kappa}$ contains the direct sum of at least c copies of $\zeta_\kappa^{N_1}$, except possibly in the exceptional cases described there, when it contains the sum of at least c copies of $\zeta_\kappa^{N_1}/\mu$, where $\mu \in \mathrm{Irr}_P(N_1)$. Observe that we use the estimate $r \geqslant 1$, but in the orthogonal case we do have $r \geqslant 2$. Let $y_{1|W_1^\perp} = \omega \cdot \mathrm{Id}$, where $\omega \in F$, and set $W' = (y_1 - \omega \cdot \mathrm{Id})V$, so that $W' \subseteq W_1$. Observe that in all the exceptional cases listed in 4.1 $y_{1|W'}$ is irreducible. We show that the assumption $k > 1$ implies that $\dim W' = 1$, and so it rules out the exceptions (a) and (e). Indeed, if $\dim W' > 1$, then $y_{1|W'}$ is of maximal order in $I(W')$ (i.e., it is a Singer cycle in $I(W')$, of order $q + 1$ in cases (a)–(d), and of order 9 in case (e)). Clearly W' is an h-module, and $h_{|W'} \subseteq I(W')$. Hence $y_{1|W'} = (h_{|W'})^k$, and moreover $h_{|W'}$ lies in a Singer cycle of $I(W')$. Obviously this implies that $h_{|W'}$ and $y_{1|W'}$ have the same order. But this is impossible if $k > 1$, since $h_{|W'}$ has order s, while $y_{1|W'}$ has order l modulo the scalars.

Next, suppose that $\dim W' = 1$. Let λ be the order of $y_{1|W'}$. By the above, we may assume that λ is a proper divisor of $q + 1$. Therefore, as $|y_1| = q + 1$ by Proposition 4.1, $|\omega| = q+1$. As $(h_{|W'^\perp})^k = \omega \cdot \mathrm{Id}$, it follows that $s = |h| = k(q+1)$. Let X be an irreducible h-submodule of $W'^\perp$. Again, $(h|_X)^k = \omega \cdot \mathrm{Id}$. By Schur's lemma, $\langle h|_X \rangle$ is a field, K say, and $\dim X = K : F$. Then $K \cong F(\varepsilon)$, where $\varepsilon^k = \omega$. As above, set $s = v^\gamma$, v a prime. Since the multiplicative group of F is of order $(q - 1)(q + 1)$, one observes that F contains no v^2-root of ω, and contains a v-root of ω if and only if $v = 2$. So $\dim X$ equals k if $v > 2$ and equals $k/2$ otherwise. In addition, if $v = 2$ then X is totally isotropic. (If $k = 2$ then $\dim X = 1$ and $U(1, q)$ has no elements of order $2(q + 1)$; if $k > 2$ then $k/2$ is even and

$U(k/2,q)$ contains no irreducible cyclic subgroups.) Therefore, when $v=2$, $W'^{\perp}$ contains a dual h-submodule X' such that $X+X'$ is non-degenerate of dimension k. It follows that $W'^{\perp}$ is a direct sum of submodules of dimension k, whence $\dim W'^{\perp} \equiv 0 \pmod{k}$. So $m \equiv 1 \pmod{k}$. If v is odd, then $m > k+1$, as otherwise X is non-degenerate and h stabilizes no isotropic subspace of V. □

Proposition 5.5. *Assume that $k>1$ and $(*)$ does not hold. Let $\zeta \in \mathrm{Irr}_P(N_0)$ be such that $\tilde{\theta}_{|N_0} = \zeta \cdot \mathrm{Id}$ and let c be as in Lemma 5.4. Then $\tilde{\theta}_{|G}$ contains at least c copies of the induced representation ζ^G, unless the case (2) of Lemma 5.4 holds and $\tilde{\theta}_{|G}$ contains at least c copies of a quotient ζ^G/μ^G, where $\mu \in \mathrm{Irr}_P(N_1)$.*

Proof. Recall (cf. the remark following Lemma 3.4) that $Z(\widetilde{S}) = Z(\widetilde{H})$, unless either $d=2$, $\tau=1$ and $\varepsilon=1$, or $d=1$, and $H=U(m,2)$, $SU(m,2)$, $Sp(m,2)$, $Sp(m,3)$, $O(m,2)$, $\Omega(m,2)$. However, the latter exceptional cases are ruled out by the current assumptions that U is non-abelian and $k>1$. Moreover, since we are assuming that $(*)$ does not hold, by Lemma 3.7, every non-central normal subgroup of S contains U'. Therefore, since ϕ is non-trivial on U', $\ker\phi \subseteq Z(\widetilde{S}) \subseteq Z(\widetilde{H}) \times Z(U)$. It follows easily that the order of g modulo $Z(\widetilde{H})$ is the same as the order of $g_{|T}$ modulo $Z(\widetilde{H})_{|T}$, so the latter equals $s=kl$. By Lemma 5.4, $N_{1|T_\kappa}$ contains a direct sum of at least c copies of ζ^{N_1} or ζ^{N_1}/μ, according to case (1) or (2). By Lemma 5.3, g has an orbit O' of length k on the components T_κ. Hence we may apply Proposition 2.14(i) to $T' = \bigoplus T_\kappa$ ($\kappa \in O'$), and conclude that $G_{|T}$ contains a direct sum of at least c copies of ζ^G or ζ^G/μ^G, according to case (1) or (2) in Lemma 5.4. □

Remark. Obviously, the degree of the minimum polynomial of g in the representation ζ^G/μ^G is equal to $\deg\zeta^G(g) - \deg\mu^G(g) = s-k$.

Acknowledgments

The authors are thankful to Dr. A. Zavarnitsine and the referee for useful comments on a previous version of this corrigendum.